

**COMSATS University, Islamabad**

**Islamabad Campus**

**Department of Computer Science**

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| **Assignment No. 2:**  **Formal logic Proofs and Reasoning** | |
| **Course code and Title:** CSC102, DISCRETE STRUCTURES | |
| **Instructor:** | Mr. Khurram Iqbal |
| **Assigned Date: October 23,2024** | **Due Date: November 15,2024** |
| **CLO-2:** **Apply formal logic proofs and reasoning to construct a sound argument.** | |
| **Instructions:**   1. Try to get the concepts, consolidate your concepts and ideas from these questions. 2. You should consider **recommended books** for clarifying your concepts as handouts are not sufficient. 3. **Try to make the solution by yourself and protect your work from other students. If I found the solution files of some students are same then I will reward zero marks to all those students.** 4. Deadline for this assignment is **November 15,2024.** This deadline will not be extended. | |

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| **Question # 1(Rule of Inference/Logic Reasoning) 10 Marks** |
| * + 1. Formulate the following arguments symbolically and determine whether each is valid. Let   *p*: *I study hard. q*: *I get A’s. r*: *I get rich.*  **1.** If I study hard, then I get A’s.  I study hard.  ∴ I get A’s.  **2.** If I study hard, then I get A’s.  If I don’t get rich, then I don’t get A’s.  ∴ I get rich  **3.** I study hard if and only if I get rich.  I get rich.  ∴ I study hard.  **4.** If I study hard or I get rich, then I get A’s.  I get A’s.  ∴ If I don’t study hard, then I get rich.  **5.** If I study hard, then I get A’s or I get rich.  I don’t get A’s and I don’t get rich.  ∴ I don’t study hard.   * + 1. Write the given argument in words and determine whether each argument is valid. Let   *p*: *4 gigabytes is better than no memory at all.*  *q*: *We will buy more memory.*  *r* : *We will buy a new computer.*      4. * 1. For each set of premises below, apply the relevant rules of inference to derive the logical conclusion(s). 5. “If I take the day off, it either rains or snows.” “I took Tuesday off or I took Thursday off.” “It was sunny on Tuesday.” “It did not snow on Thursday.” 6. “If I eat spicy foods, then I have strange dreams.” “I have strange dreams if there is thunder while I sleep.” “I did not have strange dreams.” 7. “I am either clever or lucky.” “I am not lucky.” “If I am lucky, then I will win the lottery.”    * 1. For each of the following arguments, identify the specific rule of inference being applied. Justify your choice by explaining how the rule is used to derive the conclusion from the given premises. 8. Ali is a mathematics major. Therefore, Ali is either a mathematics major or a computer science major. 9. Javed is a mathematics major and a computer science major. Therefore, Javed is a mathematics major. 10. If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed. 11. If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today. 12. If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn. |
| **Question # 2(Proof Methods) 10 Marks** |
| 1. Using the definitions of even integer and odd integer, give a ***direct proof*** that this statement is true for all integers n: If n is odd, then 5n + 3 is even. 2. Using the definitions of even integer and odd integer, give a ***proof by contraposition*** that this statement is true for all integers n: If 3n − 5 is even, then n is odd. 3. Prove that following given statement is true for all integers n, using the definitions of even integer and odd integer: If 7n − 5 is odd, then n is even. 4. Suppose a, b, and c are odd integers. Prove that the roots of a*x*2 + b*x* + c = 0 are not rational. 5. Give a ***proof by contradiction*** of: “If n is an even integer, then 3n + 7 is odd.” 6. Prove that this statement is true for all integers n: n is odd if and only if 5n + 3 is even. 7. Show that the statement “Every integer is less than its cube” is false by finding a counterexample. 8. Prove that there is only one pair of positive integers that is a solution to 3x2 + 2y2 = 30. 9. Prove that the square of every even integer ends in 0, 4, or 6. 10. Prove that the following is true for all real numbers x and y: max(x, y) = 1/2 (x + y + |x − y|). |
| **Question # 3(Mathematical Induction) 10 Marks** |
| 1. Use mathematical induction to prove that        1. Use mathematical induction to prove that     for all integers n ≥2   1. Prove by mathematical induction     for all integers n≥1   1. Use mathematical induction to prove the generalization of the following De Morgan’s Law:     where A1, A2, …, An are subsets of a universal set U and n≥2.   1. Use mathematical induction to prove that n3 - n is divisible by 3 whenever n is a positive integer. 2. Use mathematical induction to prove that for all integers n≥1, 22n-1 is divisible by 3. 3. Use mathematical induction to show that the product of any two consecutive positive integers is divisible by 2. 4. Prove by mathematical induction n3 - n is divisible by 6, for each integer n ≥ 2. 5. Prove by mathematical induction. For any integer n ≥ 1, xn - yn is divisible by x - y, where x and y are any two integers with x ≠ y. 6. Use mathematical induction to prove that for all integers n ≥ 3.   2n + 1 < 2n |